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Observational constraint on the fourth derivative of the inflaton potential

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ABSTRACT

We consider the flow equations for the three slow-roll parameters n_S (scalar spectral index), r (tensor-to-scalar ratio), and $dn_S/d \ln k$ (running of the spectral index). We show that the combination of these flow equations with the observational bounds from cosmic microwave background and large-scale structure allows one to put a lower bound on the fourth derivative of the inflationary potential, $M_P^4(V''''/V) > -0.02$.

Key words: cosmic microwave background – large-scale structure of Universe.

1 INTRODUCTION

Inflation provides the creation of scalar, vector and tensor perturbations in the metric; the scalar ones may be the origin of the formation of large-scale structures, the vector ones decayed away, and the tensor ones gave rise to a stochastic background of gravitational waves. It is possible to extract information about the features of the spectra of these primordial perturbations from observations of the cosmic microwave background (CMB) anisotropies, or from measurements of the matter power spectrum from large-scale structure (LSS). The observable variables are the power spectral indices of the density and tensor perturbations, n_S and n_T , and the overall amplitude of these, usually denoted by S and T . The knowledge of these quantities allows us first of all to test the inflationary scenario, by checking if the theoretically predicted consistency relation is satisfied by the observational values; and then, eventually, to reconstruct the scalar-field potential. The usual way to express these spectra is by a Taylor expansion in the deviation from scale invariance, which can be directly related to the slow-roll expansion in the inflaton potential (Lindsey et al. 1997; Martin, Riazuelo & Schwarz 2000; Leach et al. 2002). Each observable quantity can then be related to the parameters in the slow-roll expansion.

The evolution of the observables during inflation can be followed by the flow equations (Hoffman & Turner 2001), and in Hoffman & Turner (2001) it was found, that the lines $T/S \simeq 0$ and $T/S \simeq -5(n_S - 1)$ act as attractors for the evolution in the $(n_S, T/S)$ plane. This result provides a new relation between the variables n_S and T/S , which the authors suggest can be used as a second consistency relation in interpreting CMB results. A subsequent analysis found (Hansen & Kunz 2002) that by combining the observational bounds with the flow equations, a non-trivial constraint on the value of the third derivative of the inflaton potential can be obtained. Recently Kinney (2002) generalized the inflationary flow equations to arbitrary order in slow roll, and numerically integrated them after a truncation at the fifth order. In this way one can consider the clustering of points in the tridimensional parameter space, including the

running of the spectral index, and hence generalizing the results of Hoffman & Turner (2001).

In this Paper we will extend the analyses of Hoffman & Turner (2001), Hansen & Kunz (2002) and Kinney (2002), and show that a combination of the higher order flow equations and the observational bound on the inflationary parameters allows one to put a non-trivial constraint on the fourth derivative of the inflaton potential.

2 THE FLOW EQUATIONS

Slow-roll models are traditionally defined through the three parameters ϵ , η and ξ^2 , which are related to the first, second, and third derivative of the inflaton potential with respect to the inflaton field ϕ (to simplify the expression we avoid write the reduced Planck mass, $M_P = 2.4 \times 10^{18}$ GeV)

$$\epsilon \equiv \frac{1}{2} \left(\frac{V'}{V} \right)^2, \quad \eta \equiv \frac{V''}{V}, \quad \xi^2 \equiv \frac{V'V'''}{V^2}. \quad (1)$$

Assuming that ϵ and η satisfy the flatness conditions, $\epsilon \ll 1$ and $|\eta| \ll 1$, one finds that the scalar spectral index n_S , the tensor to scalar ratio r , and the running of the spectral index $\partial_{\ln k} \equiv (dn_S/d \ln k)$, all observable quantities, can be expressed in terms of the slow-roll parameters as

$$\begin{aligned} n_S - 1 &= 2\eta - 6\epsilon + O(\xi^2), \\ n_T &= -\frac{r}{\kappa} = -2\epsilon + O(\xi^2), \\ \partial_{\ln k} &= -2\xi^2 - 24\epsilon^2 + 16\epsilon\eta + O(\sigma^3), \end{aligned} \quad (2)$$

with the definition

$$\sigma^3 = \frac{V'^2 V'''}{V^3},$$

where the factor κ depends on the given cosmology (Knox 1995; Turner & White 1996), in particular on the value of Ω_Λ and Ω_M . In this Paper we will use the value $\kappa = 5$, corresponding to $\Omega_\Lambda = 0.65$ and $\Omega_M = 0.35$.

The first two expressions in equation (2) are truncated at order ξ^2 , ignoring errors that are quadratic in ϵ and η , and requiring

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that $|\xi^2| \ll \max(\epsilon, |\eta|)$ (Lyth & Riotto 1999). The third expression shows us that every time we take the derivative of a slow-roll parameter we get a quantity which is one order higher in slow roll, and we therefore need to introduce the new parameter $|\sigma^3| \ll \max(\epsilon^2, \epsilon|\eta|, |\xi^2|)$.

From the set of equations (2) one can derive a system of differential equations which describes the evolution of n_S and r as the inflaton rolls down its potential, and is only weakly dependent on the form of the potential itself. By taking, at first order, $(d\phi/d\ln k) = \sqrt{2\epsilon}$, where ϕ is the inflaton field, one gets (Liddle & Lyth 1992; Kosowsky & Turner 1995)

$$\frac{dn_S}{d\ln k} = 4\frac{r}{\kappa} \left[(n_S - 1) + \frac{3}{2}\frac{r}{\kappa} \right] - 2\xi^2, \quad (3)$$

$$\frac{dr}{d\ln k} = \frac{r}{\kappa} \left[(n_S - 1) + \frac{r}{\kappa} \right]. \quad (4)$$

In our attempt to find a constraint on the quantity V''''/V , we will make the assumption that either σ^3 or V''''/V is constant (see below), so we will not need to require a better accuracy than the one given in the set of equations (2), and in the corresponding first-order relation $(d\phi/d\ln k) = \sqrt{2\epsilon}$. The second derivative of the scalar spectral index takes the form

$$\frac{d^2 n_S}{d\ln k^2} = 2\sigma^3 + 2\xi^2\eta - 24\xi^2\epsilon - 192\epsilon^3 + 192\epsilon^2\eta - 32\epsilon\eta^2 + O(\tau), \quad (5)$$

with $\tau \propto (d\sigma^3/d\ln k)$, which in our approximation is negligible or zero. By using the definitions in equation (2), we get the new system of flow equations

$$\frac{dn_S}{dN} = -\partial_{\ln k} \quad (6)$$

$$\frac{dr}{dN} = -\frac{r}{\kappa} \left[(n_S - 1) + \frac{r}{\kappa} \right] \quad (7)$$

$$\begin{aligned} \frac{d\partial_{\ln k}}{dN} = & -2\sigma^3 - \frac{1}{2}\partial_{\ln k} \left[9\frac{r}{\kappa} - (n_S - 1) \right] + 2(n_S - 1)^2 \frac{r}{\kappa} \\ & + 15(n_S - 1) \left(\frac{r}{\kappa} \right)^2 + 15 \left(\frac{r}{\kappa} \right)^3, \end{aligned} \quad (8)$$

where we have used the first-order expression $d\ln k = -dN$, consistent with our assumptions in equations (2), and N is the number of Hubble times (e-folds) until the end of inflation. The parameter σ^3 is related to the fourth derivative of the potential through $\sigma^3 = rV''''/5V$. In order to close this set of equations, we will need to assume that either σ^3 or V''''/V can be treated as a constant throughout inflation. The choice between these two assumptions is arbitrary, but will affect the behaviour of the flow in the parameter space.

The observational bounds which we will use are the ones obtained in Hannestad et al. (2002) (for similar results see Kinney, Melchiorri & Riotto 2001; Hannestad, Hansen & Villante 2001; Wang, Tegmark & Zaldarriaga 2002; Leach & Liddle 2002)

$$0.8 < n_s < 1.0, \quad (9)$$

$$0 < r < 0.3, \quad (10)$$

$$-0.05 < \partial_{\ln k} < 0.02. \quad (11)$$

Such observations can be inverted to give constraints on the inflationary potential and its derivatives. The *COBE* observations (Bunn, Liddle & White 1996) gave the first constraint on the first derivative of the inflaton potential,

$$\frac{V^{3/2}}{M^3 V'} \approx 5 \times 10^{-4}, \quad (12)$$

and the bounds above, equations (9)–(11), provide constraints on the first and second derivatives of the potential: $|V'/V| < 0.25$, $|V''/V| < 0.1$. Hansen & Kunz (2002) found $V''''/V > -0.2$.

We will study the flow in the three-dimensional parameter space $(n_S, r, \partial_{\ln k})$, and by also making use of the observational constraints (9)–(11), we will be able to induce a limit on the fourth derivative of the potential, V''''/V .

3 DISCUSSION

In slow-roll inflation, the scales relevant for structure formation crossed outside the horizon roughly 50 e-folds before the end of inflation (Kolb & Turner 1990), and it is at this time that the experimental bounds on the observable parameters in equations (9)–(11) apply. We are going to consider only the case of single field inflation, which will end when the slow-roll conditions $V'/V < \sqrt{6}$ and $V''/V < 3$ are violated. In terms of the value of the parameters n_S and r , these conditions translate into the SR ‘validity-region’ (Kolb & Turner 1990; Hoffman & Turner 2001)

$$\frac{r}{\kappa} < 6 \quad \text{or} \quad \left| (n_S - 1) + \frac{3}{2}\frac{r}{\kappa} \right| < 6. \quad (13)$$

Using the evolution equations (6)–(8), we let all the points of the boundary (13) flow back 50 e-folds (we take the value $\kappa = 5$). This is carried out for different fixed values of the parameter σ^3 (or V''''/V). We can then induce a constraint on this latter by demanding that, at $N = 50$, at least some of the points of the boundary land inside that region in the $(n_S, r, \partial_{\ln k})$ space which is allowed by observations, equations (9)–(11). If no points land inside the space allowed by observations, then we can deduce that we took an unrealistic value for the parameter σ^3 or V''''/V .

To follow the flow backwards in time, the slow-roll conditions (13) provide us with the initial values for the two variables n_S and r . As initial condition for the variable $\partial_{\ln k}$ we will take equation (3), with the extra assumption that $\xi^2 = 0$. We verified that the position of the points after 50 e-fold is virtually independent on the initial conditions chosen, and therefore this assumption does not affect our results. Moreover, with these initial conditions and in the case

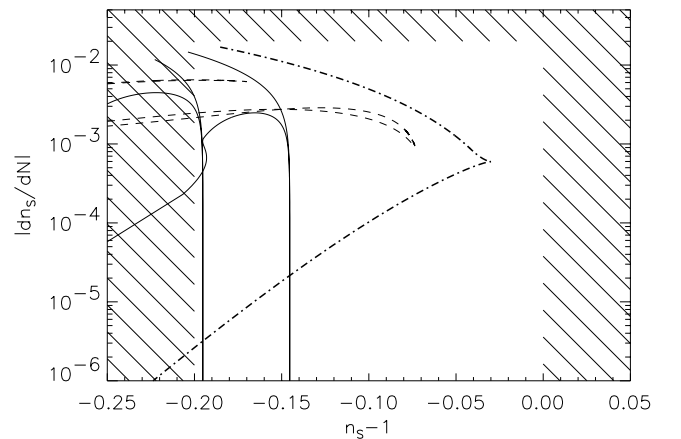


Figure 1. The SR validity region flown back 50 e-folds, for different fixed values of the parameters σ^3 and V''''/V . The dot-dashed line shows the case $\sigma^3 = 0$ (identical to $V''''/V = 0$); the two dashed lines show, from right to left, the two cases $\sigma^3 = -10^{-4}$ and $\sigma^3 = -3 \times 10^{-4}$; the solid lines instead represent the case in which V''''/V is kept constant, and the values are, again from right to left, $V''''/V = -10^{-2}$ and $V''''/V = -1.8 \times 10^{-2}$. The hatched region is excluded by observations. It is clear from the figure, that a slightly larger negative value of the parameter σ^3 (or V''''/V) will not be acceptable because it does not satisfy the observational constraint.

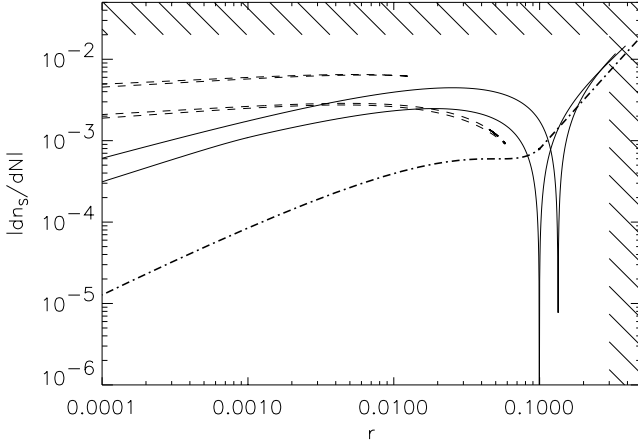


Figure 2. Same as Fig. 1, but for r and dn_s/dN .

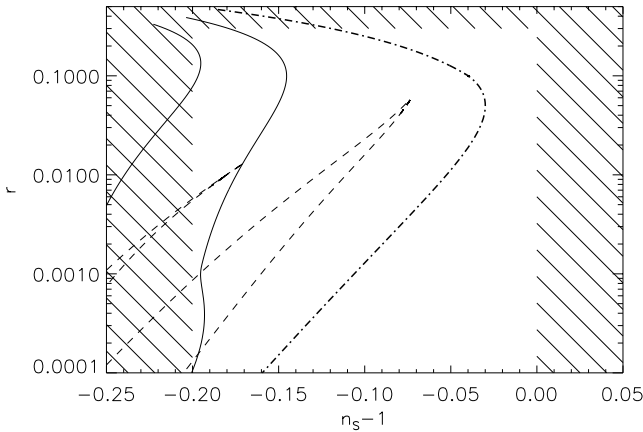


Figure 3. Same as Fig. 1, but for r and n_s .

$\sigma^3 = 0$, all the points of the boundary flow into the validity region. For other cases (e.g. $\sigma^3 \neq 0$) we exclude numerically the possibility that a point could first flow outside this validity region and then later re-enter the region.

In the case where σ^3 is treated as a constant during the last 50 e-folds, we find that for $\sigma^3 < -3.5 \times 10^{-4}$ there are no final points in agreement with observations. This is easily seen in Fig. 1, in which the dashed line which corresponds to the value $\sigma^3 = -3 \times 10^{-4}$ is about to exit from the region given by the observational limits, equation (9). On the other hand, we are not able to induce any limit

on the parameter σ^3 in the case $\sigma^3 > 0$, because for sufficiently small value of r there will always be points in the allowed region. Similarly Figs 2 and 3 include the parameter r .

If one instead assumes that V'''/V can be treated as a constant during inflation, the curve which represent the evolved validity region exceeds the observational boundary if $V'''/V < -0.02$. Also in this case there will always be acceptable points if V'''/V takes positive values, however, also in this case only for very small r .

In conclusion, we have shown that the combination of the slow-roll equations (6)–(8) and the observational results (9)–(11) leads to a lower bound $V'''/V > -0.02$, for single field models which end inflation by breaking the slow-roll conditions (13). With a larger number of e-folds this bound becomes slightly stronger.

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